High Functionality Mode Transformers with Bandwidth Control and Mode Selectivity

Ming-Chuan Yang and Kevin J. Webb
School of Electrical and Computer Engineering
Purdue University, West Lafayette, IN 47907-1285

Abstract – We propose high functionality field transformation in irregular rectangular waveguide structures. Multi-resolution optimization allows good convergence and a stepped wall profile provides for a large number of degrees of freedom. The resulting structures are compact, with dimension of a few wavelengths, and can have functionality not achievable through periodic means. For illustration, we have designed frequency-dependent mode converters, mode-selective reflectors, and multiple mode converters, where more than one mode is converted simultaneously, all with virtually 100% efficiency. Non-uniqueness affords design selection based on the frequency response.

Index Terms – Multi-resolution, irregular waveguide, mode transformation

I. Introduction

Periodic structures have been explored for decades to achieve mode transformation and filtering functions. For instance, such periodic gratings have been used as mode transformers in overmoded circular waveguides for mode transformation in high power microwave sources [1]. The design approach that has been employed uses coupled mode theory and requires weak scattering [2]. This provides few degrees of freedom and the overall length of the mode converter is dictated by the beat wavelength between the input and output mode. Another example is the periodic dielectric stack filter or Bragg grating. Treating the layer parameters as variables, the dielectric stack filter problem has been studied using genetic-based optimization [3]. In diffractive optics, the weakly scattering binary phase plate has been treated as an optimization problem [4].

Using a totally different approach that relies on strong scatter, compact and highly efficient irregular waveguide mode transformation structures have been proposed, in which complete mode conversion was achieved within several wavelengths [5], [6]. These elements were composed of optimized variations in the conducting waveguide wall for 2-dimensional parallel-plate and 3-dimensional circular waveguide structures. Stepwise scattering sections were treated as optimization variables, and the resulting design appeared as a cascaded series of irregular steps. In this work, the optimization algorithm involved varying the step size, the step length, and the overall length of the structure using decreasing step size and a simple optimization routine that sequentially updated a small subset of the variables [7]. A circular waveguide TE_{11} to TM_{11} mode converter built for operation at X-band validated the approach [6].

Here we propose a multi-resolution synthesis algorithm that couples a forward mode matching solver to a MATLAB optimizer, and apply this optimization approach to the design of several unique rectangular waveguide mode transformers. Our approach has shown that the large number of degrees of freedom not only yields high functionality, but also the possibility of more than one solution. This enables us to have more freedom in controlling the bandwidth response or other physical properties, given that there is a specific mode transformation requirement at a certain frequency. Moreover, the computation time was quite modest, allowing designs to be reached within several hours on an AMD Athlon machine with a Linux platform. The functionality of the example designs of frequency-dependent mode converters, mode-selective reflectors, and multiple mode converters establish the functional importance of the approach.

II. Multi-Resolution Synthesis

The multi-resolution approach we employ propagates coarse step solutions to finer steps. This approach significantly enhances the convergence properties of the nonlinear optimization problem of synthesizing a stepwise scattering structure to achieve a particular mode transformation. The refinement from coarse to fine helps avoid local minima, and evaluation with several random coarse initial solutions establishes a starting vector for the algorithm.

The general concept of our rectangular waveguide structure is depicted in Fig. 1(a), in which a uniform height (h) rectangular waveguide has stepwise variations in width to achieve mode transformation. The lengths of the input and output waveguides are assumed to be infinite. We synthesize the scattering surface by exciting a certain field distribution on one side and then optimizing the wall profile until the desired transformed field is achieved. The TE_{10} mode is the fundamental mode in the structures we consider, and there is no coupling to TM modes. The length (l) and width (w) of each section are used as optimization parameters in our simulation, as shown in Fig. 1(b). We achieve transformation within several wavelengths by optimizing the width and length $(w_1, w_2, ..., w_n, l_1, l_2, ..., l_n)$, once the height and the total length of the structure are chosen. Here we set the length of each section equal.

We choose the cost function

$$P = -P_{mode_1,f_1} P_{mode_2,f_2} P_{mode_3,f_3} \cdots P_{mode_M,f_M} \qquad (1)$$

0-7803-7239-5/02/$10.00 © 2002 IEEE

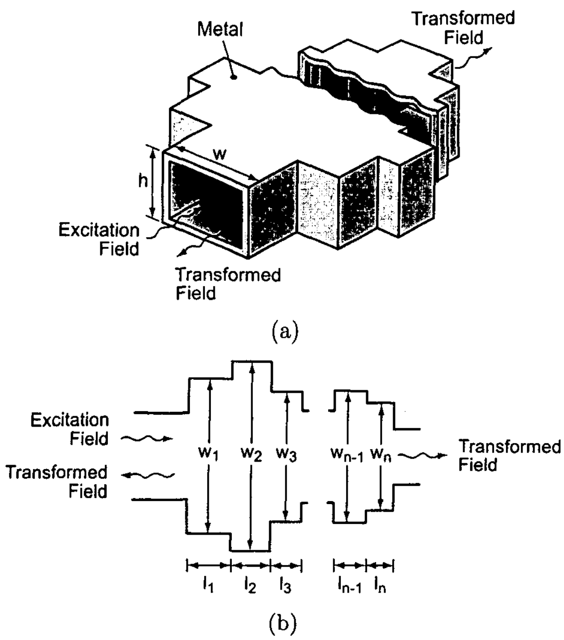

Fig. 1. Schematic diagram of (a) the irregular scattering waveguide structure and (b) the parameters to be optimized to achieve complete field transformation.

where P_{mode_m,f_m} is the normalized power of the mth mode at the mth frequency, with m ranging from 1 to M, and M is the number of modal powers to be maximized. The negative sign is added to the right side of (1) because we use the MATLAB minimizing function *fmincon.* The mode power may be measured on transmission or reflection. We choose our cost function as a direct multiplication rather than addition so that the information of some P_{mode_m,f_m} dominates the value of the cost function if all the other quantities have been optimized. The cost function of (1) incorporates the power of several modes at different frequencies. However, it can be reformulated so that other properties, such as phase information, can be included.

The forward solution is achieved using the mode-matching method [8] with generalized scattering matrix cascading [9]. The code for this solver was written in C^{++} and is called from MATLAB.

We start the synthesis procedure using randomly-generated widths for a coarse 5-section transformer. Since the initial vector is not guaranteed to be a good starting point for the optimization problem, we use a reduced eigenmode set for fast, approximate evaluation. Several random vectors are evaluated with a threshold transformation efficiency (10% was found to be a suitable value). After we optimized the coarse scattering surface, with the resolution of the initial vector, we further divide each section into two subsections, which doubles the number of stepwise sections, and then re-optimize. This refinement process continues through a predetermined number of levels. The minimum step-size for the formation of the finite-difference gradient and the tolerance for the stopping criterion are adjusted, depending on the resolution. The number of modes in each section for each forward solution is large enough for convergence. After several steps of the optimization, an almost complete field transformation is reached and the related scattering surface is obtained. Based on this algorithm, we can usually find solutions with efficiencies of field transformation higher than 99%.

III. Example Mode Transformers

A. Frequency-Dependent Mode Transformer

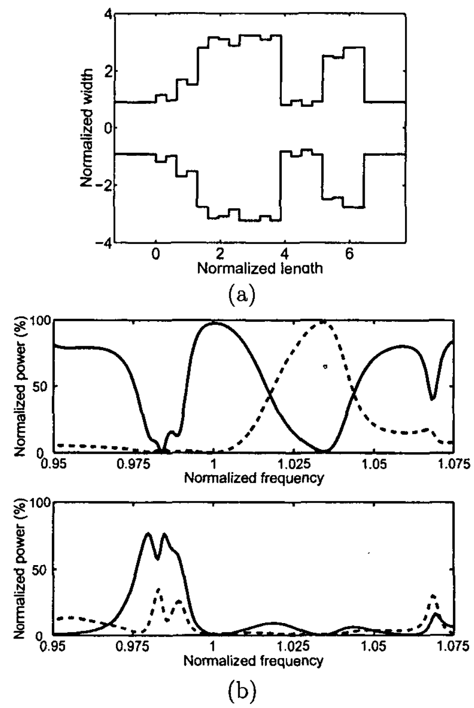

Fig. 2. (a) A frequency-dependent mode converter. The TE_{10} mode is excited on the left side. (b) In the upper graph, solid: transmitted TE_{10}; dashed: transmitted TE_{30}. In the lower graph, solid: reflected TE_{10}; dashed: reflected TE_{30}.

Fig. 2(a) shows an example of a frequency-dependent structure that allows a TE_{10} mode incident from the left at f_0 to pass without loss, while a TE_{10} mode incident at $1.03f_0$ is almost totally transformed to a TE_{30} mode at the output. The length scales (for this and all transformers presented) are normalized to $\lambda_0 = c/f_0$. The far left and right widths are selected so that only TE_{10} and TE_{30} propagate (symmetry prevents TE_{20} being excited). The upper graph in Fig. 2(b) shows the total transmission for TE_{10} at f_0 and TE_{30} at $1.03f_0$, while the lower graph, showing the reflected power of each mode on the left side of the structure as a function of frequency, indicates essentially no reflected wave at these frequencies. The total power, if the four curves are summed, is unity at every frequency, satisfying conservation of energy. Another structure with the same functionality is shown in Fig. 3. While the same field transformation is achieved at f_0 and $1.03f_0$, the frequency response is very different, therefore providing a choice based on frequency performance. Also, note that the total length of each converter is about $6.45\lambda_0$.

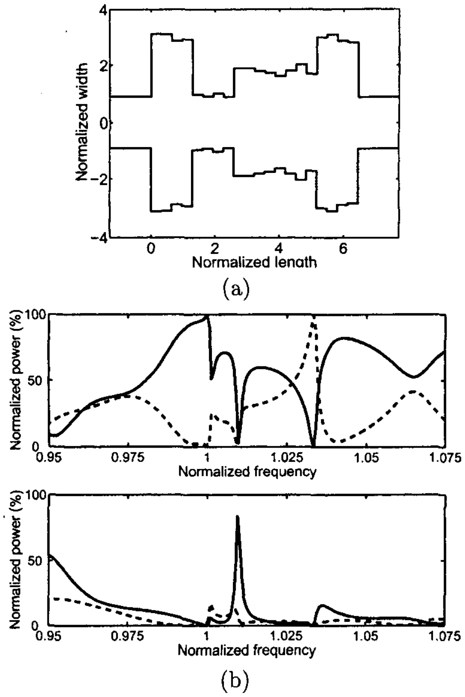

Fig. 3. (a) Another frequency-dependent mode converter. The TE_{10} mode is excited on the left side. (b) In the upper graph, solid: transmitted TE_{10}; dashed: transmitted TE_{30}. In the lower graph, solid: reflected TE_{10}; dashed: reflected TE_{30}.

B. Mode-Selective Reflector

A mode-selective reflector design is shown in Fig. 4(a), where the TE_{10} mode incident from the left is totally reflected at f_0, while TE_{20} incident at f_0 is unattenuated. The widths on the far left and right sides are chosen so that only TE_{10} and TE_{20} modes will propagate, but symmetry with TE_{10} incident precludes coupling to TE_{20}. In Fig. 4(b), the upper graph shows the transmitted power of TE_{10} approaching zero at f_0, where the transmitted power of TE_{20} reaches its maximum value. The frequency response of the reflected power of each mode is shown in the lower graph of Fig. 4(b). Another example is shown in Fig. 5, where this design has a smoother frequency response with about double the bandwidth. The total length of these structures is only about $1.6\lambda_0$.

C. Multi-Mode Transformation

Figure 6(a) shows an example of an in-phase $TE_{10}/TE_{30} \rightarrow TE_{10}$ mode transformer. Equal power in the in-phase (at the zero length position) input TE_{10} and TE_{30} modes is assumed from the left. The upper graph of Fig. 6(b) shows that complete field transformation is reached for the

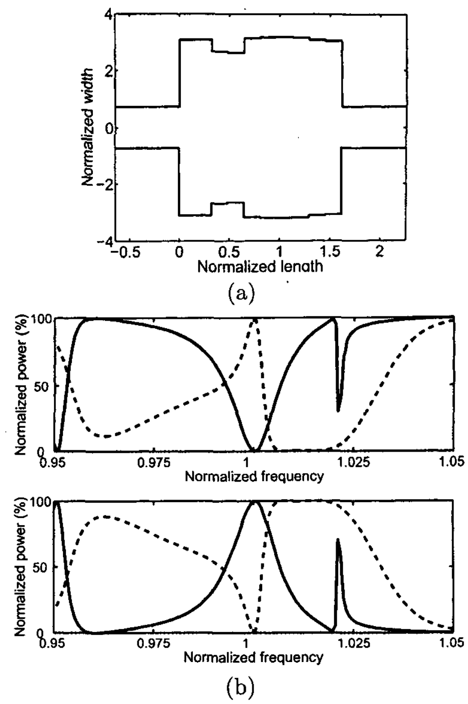

Fig. 4. (a) A mode-selective reflector. The TE_{10} or the TE_{20} mode is excited on the left side. (b) Upper graph: solid: transmitted TE_{10}; dashed: transmitted TE_{20}. Lower graph: solid: reflected TE_{10}; dashed: reflected TE_{20}.

transmitted TE_{10} mode (solid line), where the reflected TE_{10} mode (dashed line) and TE_{30} mode (dotted line) are also shown for reference. To prove reciprocity, we then excite the TE_{10} mode on the right side of the structure. The lower graph of Fig. 6(b) shows that there is an equal power splitting of TE_{10} (solid line) and TE_{30} (dashed line) modes appearing on the left side of the structure at the frequency where the transmitted TE_{10} mode reaches its maximum in the upper graph.

IV. Conclusion

We have proposed a way to search for the optimized irregular scattering surface of rectangular waveguide structures to achieve complete field transformation. The approach is based on a multi-resolution algorithm together with a local minimum optimizer. Due to the large number of degrees of freedom, for the structures studied, we can usually find more than one solution to achieve the desired functionality with complete field transformation. Our results show that there are adequate degrees of freedom for some control of the frequency response and other physical properties, thereby leading to better and more sophisticated control of the field transformation. This may lead to useful structures involving multiple mode transformation for sources, power combiners and splitters, phase shifters, and couplers. Fabrication at microwave frequencies can employ commonly used metal waveguide or litho-

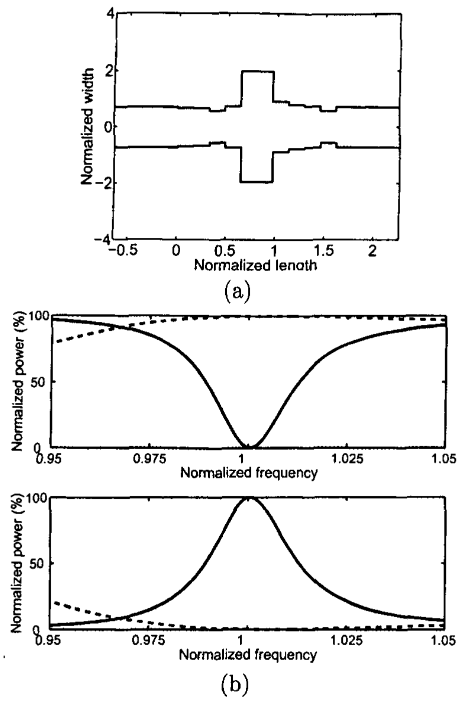

Fig. 5. (a) Another mode-selective reflector. The TE_{10} or the TE_{20} mode is excited on the left side. (b) In the upper graph, solid: transmitted TE_{10}, dashed: transmitted TE_{20}. In the lower graph, solid: reflected TE_{10}; dashed: reflected TE_{20}.

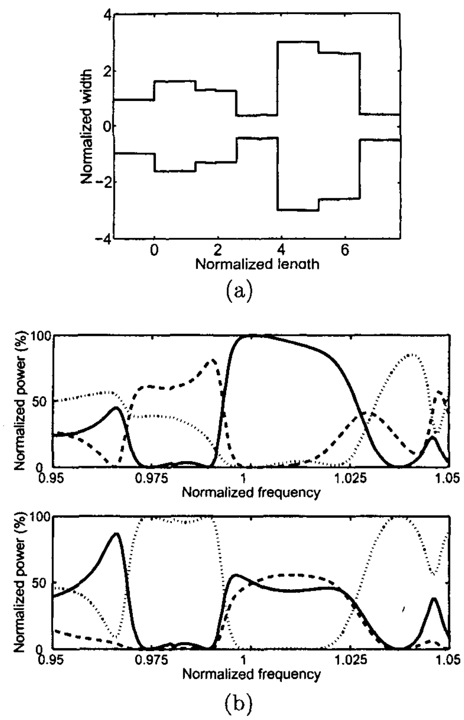

Fig. 6. (a) A multi-mode converter. (b) In the upper graph, the composite TE_{10}/TE_{30} mode is excited on the left side. Solid: transmitted TE_{10}; dashed: reflected TE_{10}; dotted: reflected TE_{30}. In the lower graph, the TE_{10} mode is excited on the right side. Solid: transmitted TE_{10}; dashed: transmitted TE_{30}; dotted: reflected TE_{10}.

graphic techniques. At terahertz and optical frequencies, semiconductor processing, as for micro-machines, could be employed.

V. Acknowledgment

This work was supported by ARO under contract DAAD 19-00-1-0387 and NSF under contract 0121317-ECS.

References

[1] M. Thumm, "High-power millimetre-wave mode converters in overmoded circular waveguides using periodic wall perturbations," *Int. J. Electronics*, vol. 57, no. 6, pp. 1225–1246, 1984.

[2] N. F. Kovalev, I. M. Orlova, and M. I. Petelin, "Wave transformation in a multimode waveguide with corrugated walls," *Radio Phys. Quant. Electron.*, vol. 11, no. 5, pp. 783–786, 1968.

[3] E. Michielssen, "Optimal multilayer filter design using real coded genetic algorithms," *IEE Proc. J*, vol. 139, pp. 413–420, Dec. 1992.

[4] D. W. Prather, J. N. Mait, M. S. Mirotznik, and J. P. Collins, "Vector-based synthesis of finite aperiodic subwavelength diffractive optical elements," *J. Opt. Soc. Am. A*, vol. 15, no. 6, pp. 1599–1607, Jun. 1998.

[5] T. Haq, K. J. Webb, and N. C. Gallagher, "Optimized irregular structures for spatial- and temperal-field transformation," *IEEE Trans. Microwave Theory Tech.*, vol. 46, no. 12, pp. 1856–1867, Nov. 1998.

[6] T. Haq, K. J. Webb, and N. C. Gallagher, "Compact circular waveguide mode converters," *Microwave Opt. Technol. Lett.*, vol. 13, no. 5, pp. 251–255, Dec. 1996.

[7] T. Haq, K. J. Webb, and N. C. Gallagher, "Scattering optimization method for the design of compact mode converters for waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 3, pp. 559–565, Mar. 1995.

[8] A. Wexler, "Solution of waveguide discontinuities by modal analysis," *IEEE Trans. Microwave Theory Tech.*, vol. 15, no. 9, pp. 508–517, Sep. 1967.

[9] T. S. Chu and T. Itoh, "Generalized scattering matrix method for analysis of cascaded and offset microstrip step discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. 34, no. 2, pp. 280–284, Feb. 1986.